Integrali elementari o immediatamente riconducibili a integrali elementari

$f(x)$	$\int f(x)\mathrm{dx}$
x^3	$\frac{x^4}{4}+C$
2^x	$\frac{2^x}{\ln 2}+C$
$\frac{1}{3+3x^2}$	$\frac{1}{3}\arctan x+C$

Integrali risolvibili per scomposizione

$f(x)$	$\int f(x)\mathrm{dx}$
$\frac{3}{\sqrt{1-x^2}}-\frac{2}{1+x^2}$	$3\arcsin x-2\arctan x+C$
$\frac{\cos 2x}{senx-\cos x}$	$-senx+\cos x+C$
$2\sin x+\frac{\sin 2x}{\sin x}$	$-2\cos x+2\sin x+C$
$e^{3x}+\frac{5}{x}$	$\frac{e^{3x}}{3}+5\ln\|x\|+C$
$\frac{2x^4-3x^2+5x}{x^2}$	$\frac{2}{3}x^3-3x+5\ln\|x\|+C$
$\frac{x^3-4x^2+3}{x}$	$\frac{x^3}{3}-2x^2+3\ln\|x\|+C$
$\frac{3x^5-2x^3+1}{x^3}$	$x^3-2x-\frac{1}{2x^2}+C$

Integrali risolvibili per sostituzione lineare $t=j(x)=ax+b$

$f(x)$	t	$\int f(x)\mathrm{dx}$
$\sin(3x-5)$	$3x-5$	$-\frac{1}{3}\cos(3x-5)+C$
$(x-6)^5$	$x-6$	$\frac{(x-6)^6}{6}+C$
$(2x+5)^3$	$2x+5$	$\frac{(2x+5)^4}{8}+C$
e^{5x-1}	$5x-1$	$\frac{e^{5x-1}}{5}+C$

$\cos 5x$	$5x$	$\frac{\sin 5x}{5}+C$
$\frac{1}{(2x-3)^3}$	$2x-3$	$-\frac{1}{4(2x-3)^2}+C$
$\frac{1}{\sqrt{1-4x^2}}$	$2x$	$\frac{1}{2}\arcsin 2x+C$

Integrali risolvibili per sostituzione generica

$f(x)$	t	$\int f(x)\mathrm{dx}$
$\sin x\cos x$	$\sin x$	$\frac{\sin^2 x}{2}+C$
$\frac{2x-3}{(x^2-3x+1)^2}$	x^2-3x+1	$-\frac{1}{x^2-3x+1}+C$
$\frac{\cos x}{\sin^3 x}$	$\sin x$	$-\frac{1}{2\sin^2 x}+C$
$\frac{\ln^3 x}{x}$	$\ln x$	$\frac{\ln^4 x}{4}+C$
$\frac{\sqrt[3]{\ln x}}{x}$	$\ln x$	$\frac{1}{4}\ln x\sqrt[3]{\ln x}+C$
$\frac{1-\cos x}{(x-\sin x)^2}$	$x-\sin x$	$-\frac{1}{x-\sin x}+C$
$\frac{x^2}{x^3-9}$	x^3-9	$\ln\|x^3-9\|+C$
$\frac{x-1}{1+x^2}$	$1+x^2$ (*)	$\frac{\ln(1+x^2)}{2}-\arctan x+C$

(*) si spezza in $f(x)=\frac{x}{1+x^2}-\frac{1}{1+x^2}$ e la sostituzione si applica solo al primo addendo mentre il secondo addendo è immediato

$x^2\sqrt[3]{x^3-4}$	x^3-4	$\frac{1}{4}(x^3-4)\sqrt[3]{x^3-4}+C$
$e^{2x}\sqrt{1+e^{2x}}$	$1+e^{2x}$	$\frac{1}{3}(1+e^{2x})\sqrt{1+e^{2x}}+C$
$\frac{e^x-e^{-x}}{e^x+e^{-x}}$	e^x+e^{-x}	$\ln(e^x+e^{-x})+C$
$\frac{\sin\ln x}{x}$	$\ln x$	$-\cos\ln x+C$

Integrali risolvibili tramite integrazione per parti

$f(x)g'(x)$	$f(x)$	$\int f(x)g'(x)\mathrm{dx}$
$x\ln x$	$\ln x$	$\frac{x^2}{4}(2\ln x-1)+C$
$x\sin x$	x	$-x\cos x+\sin x+C$
xe^x	x	$e^x(x-1)+C$
$\ln x=1\cdot\ln x$	$\ln x$	$x(\ln x-1)+C$
$x^2\ln x$	$\ln x$	$\frac{1}{9}x^3(3\ln x-1)+C$
xe^{2x}	x	$\frac{1}{4}e^{2x}(2x-1)+C$
$x^2\sin x$	x^2	$-x^2\cos x+2\int x\cos x\,\mathrm{dx}=$ $-x^2\cos x+2(x\sin x+\cos x)+C$
$x\cos x$	x	$x\sin x+\cos x+C$
$\sin^2 x$	$\sin x$	$\int\sin^2 x\,\mathrm{dx}=-\sin x\cos x+\int\cos^2 x\,\mathrm{dx}=$ $=-\sin x\cos x+x-\int\sin^2 x\,\mathrm{dx}\Rightarrow$ $\int\sin^2 x\,\mathrm{dx}=\frac{-\sin x\cos x+x}{2}+C$

A richiesta di uno studente: $\int\sin^4 x\,\mathrm{dx}$.

Ponendo $f(x)=\sin^3 x$ e $g'(x)=\sin x$ da cui $f'(x)=3\sin^2 x\cos x$ e $g(x)=-\cos x$ ricaviamo:

$$\begin{aligned}\int\sin^4 x\,\mathrm{dx}&=-\cos x\sin^3 x+\int 3\sin^2 x\cos^2 x\,\mathrm{dx}=\\&=-\cos x\sin^3 x+\int 3\sin^2 x\left(1-\sin^2 x\right)\mathrm{dx}=\\&=-\cos x\sin^3 x+3\int\sin^2 x\,\mathrm{dx}-3\int\sin^4 x=\\&=-\cos x\sin^3 x+3\frac{-\sin x\cos x+x}{2}-3\int\sin^4 x\,\mathrm{dx}\end{aligned}$$

da cui, infine:

$$\int\sin^4 x\,\mathrm{dx}=\frac{1}{4}\left(-\cos x\sin^3 x+3\frac{-\sin x\cos x+x}{2}\right)+C$$